Rounding Functions Problem Solutions

1. Some function [] rounds DOWN a number to the nearest integer. For example [1.5]=1, [2]=2, [-1.5]=-2, ...

   Thus [-1.6]+[3.4]+[2.7]=-2+3+2=3.

   Answer: A.

2. From above we have that $17 \le \sqrt{n} < 18$ --> $289 \le n < 324$. Thus n can take 35 integer values from 289 to 323, inclusive.

   Answer: C.

3. [z] is the greatest integer less than or equal to z means that some function [] rounds DOWN a number to the nearest integer. For example [1.5]=1, [2]=2, [-1.5]=-2, ...

   Now, since [z] = -1, then $-1 \le z < 0$ --> ANY number from this range when rounded down to the nearest integer is -1.

   Answer: D.

4. Some function [] rounds UP a number to the nearest integer. For example [1.5]=2, [2]=2, [-1.5]=-1, ...

   Hence, since [x/2] = 0, then it must be true that $-1 < \frac{x}{2} \le 0$: any number from this range when rounded up gives 0.

   $-1 < \frac{x}{2} \le 0$ --> $-2 < x \le 0$, Only option B is in this range.

   Answer: B.

5. S1: 5< k < 6 means //k// = 5. Not divisible by 2. Sufficient.
   S2: //k+2.3// = 7 means 8 > k+2.3 >= 7. therefore 5.7 > k >=4.7 and //k// = 4 or 5. Not Sufficient.

   Answer: A

6. **[x] denotes to be the least integer no less than x. Is [2d] = 0?**

   [x] denotes to be the least integer no less than x, means that some **function [] rounds UP a number to the nearest integer**, for example:
   $[1.5]=2$ since 2 is the least integer which no less than 1.5;
   $[-0.5]=0$ since 0 is the least integer which no less than -0.5;
   $[1]=1$ since 1 itself is the least integer which no less than 1;

...

(1) [d] = 0 --> $-1<d\leq 0$. Now, if $d=0$ then $[2d]=[0]=0$ but if $d=-0.5$ then $[2d]=[-1]=-1$. Not sufficient.

Or: $-1<d\leq 0$ --> $-2<d\leq 0$ --> $[2d]=0$ (if $-1<2d\leq 0$) or $[2d]=-1$ (if $-2<2d\leq -1$). Not sufficient.

(2) [3d] = 0 --> $-1<3d\leq 0$ --> $-\frac{1}{3}<d\leq 0$. Even if $d=-\frac{1}{3}$ then $[2d]=[-\frac{2}{3}]=0$ (again since 0 is the least integer which no less than -2/3). Sufficient.

Or: $-1<3d\leq 0$ --> $-\frac{2}{3}<2d\leq 0$ --> $[2d]=0$. Sufficient.

Answer: B.

7. Here x is not said to be an integer. Stem defines some function, represented by the symbol $[\,]$, as the function which **rounds down** any number to an integer value:

$[3.4]=3$, $[2]=2$, $[-7.5]=-8$, ...

Q: is $[x]=0$? Or is $0\leq x<1$?

(1) 5x + 1 = 3 + 2x --> $x=\frac{2}{3}$ --> $[\frac{2}{3}]=0$. Sufficient.

(2) 0 < x < 1 --> any x from this range will round down to 0, so $[x]=0$. Sufficient.

Answer: D.

8. Some function [] rounds UP a number to the nearest integer. For example [1.5]=2, [2]=2, [-1.5]=-1, ...

Question: is $[x]=0$? --> is $-1<x\leq 0$?

(1) -1 < x < -0.1. Sufficient.

(2) [x + 0.5] = 1 --> $0<x+0.5\leq 1$ --> $-0.5<x\leq 0.5$. Not sufficient.

Answer: A.

9. Some function [] rounds UP a number to the nearest integer. For example [1.5]=2, [2]=2, [-1.5]=-1, ...

   Question: is $[x]=0$? --> is $-1<x\leq 0$?

   (1) -1< x< 1. Not sufficient.
   (2) x < 0. Not sufficient.

   (1)+(2) -1<x<0. Sufficient.

   Answer: C.

10. Some function [] rounds DOWN a number to the nearest integer. For example [1.5]=1, [2]=2, [-1.5]=-2, ...

    Question: is $d<1$?

    (1) d = y - [y] --> if y is an integer then $[y]=y$ and $d=y-[y]=0<1$, if y is not an integer [y] is nearest integer less than y and still $d=y-[y]<1$ (for example: $y=1.5$ --> $d=1.5-[1.5]=1.5-1=0.5<1$ or $y=-1.5$ --> $d=-1.5-[-1.5]=-1.5-(-2)=0.5<1$). Sufficient.

    (2) [d] = 0 --> $0\leq d<1$. Sufficient.

    Answer: D.

11. First of all, the question stem means: for any X, we must round down to the nearest integer.
    For example: x = 4.5 ==> [x] = 4; or x = 3.333 ==> [x] = 3

    Statement 1:
    [3x] = 1 ==> 1/3 < x < 2/3 because:
    if x < 1/3 ==> 3x < 1 ==> [3x] = 0, not 1
    if x > 2/3 ==> 3x > 2 ==> [3x] greater or equal 2, not 1
    So we have: 0.333 < x < 0.666 ==> 0 < x < 1 ==> [x] = 0
    Statement 1 is sufficient.

    Statement 2:
    [2x + 1] = 2 ==> 2 < 2x + 1 < 3 ==> 1/2 < x < 1
    So we have: 0.5 < x < 1 ==> 0 < x < 1 ==> [x] = 0
    Statement 2 is sufficient.

    Hence, D is correct.